The Pumping Effect of Traveling Phase Transition in Microtubes

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Abstract

A simplified model is established to analyze the pumping effect of traveling phase transition in a microtube. The relationship between mass flow and pumping pressure is discussed. A micropump is built and experimentally investigated to verify the theoretical analysis. For a micropump with inner diameter of 200 µm, the maximum flow rate reaches 33 µl/min and the maximum pump pressure is over 20 kPa. Comparison between theoretical and experimental results shows a good agreement, which proves the rationality of the model. The model suggests that the instantaneous high pressure during evaporation is the main force driving the pump. The model can be used to predict the mass flow to a specific micropump.

Keywords: pumping effect; phase transition; micropump

1. Introduction

Microscale flow is often encountered in MEMS. Fluid in microscale flow has small thermal inertia, so it can vaporize and condense quickly. Therefore, phase transition have many important applications in MEMS[1]. Several researchers[2~5] have reported some novel valve-less micropumps with phase transition which have the advantages of no mechanical moving parts and suitability for micromotion.

Fluid flowing in a microtube can be pumped in the scanning direction by supplying electric current cyclically through the heaters in the tube. Ozaki[5] investigated the pumping effect in the phase transition micropump as schematically shown in Fig. 1. He ascribed the pumping mechanism to the large kinematic viscosity difference between liquid and gas. Jun and Kim[6] fabricated a much smaller pump of this kind (4µm in hydraulic diameter) which was surface micromachined. They concluded that the pumping force mainly came from vapor pressure and surface tension.

Fig. 1 Structure of the phase transition micropump

This paper presents a new model with which to analyze the pumping mechanism of the micropump. The model can accurately predict
the flow rate and the trend of pump pressure versus flow rate under specified condition.

2. Theoretical analysis

2.1 Physical Model

Here, we focus on the pumping process shown in Fig. 2, where electric current switches among the heaters from left to right.

In step (i), the first heater A is powered only. If the current is large enough, the liquid in the heating section will vaporize and the first bubble forms. The instantaneous high pressure due to vaporizing pushes the liquid to both sides. In step (ii), heater A is switched off and heater B is powered. The first bubble cools down quickly and condenses. At the same time, the second bubble forms in the second section. The heaters are powered step by step from left to right. When the heater at the right terminal is switched off, step (i) is repeated and a new cycle begins.

Consider the model shown in Fig. 3. One heater is powered and a bubble is formed. The instantaneous high pressure $P$ pushes the liquid in the heating section in opposite directions. The distributed masses to the pumping direction and the other one are $M_1$ and $M_2$ respectively. If the liquid mass in the heating position before evaporation is $M$, then $M=M_1+M_2$. The tube length on each side is $L_1$ and $L_2$. The pressure at outlet and inlet is $P_1$ and $P_2$ respectively.

2.2 Mass flow-pressure gradient relationship

Simplify the model further and consider time-dependent flow in a tube. At $t=0$, the incompressible fluid velocity is zero. When a sudden pressure causes the fluid to flow, the momentum equation can be expressed as

$$ \frac{\partial u}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right), \quad (1) $$

with initial and boundary conditions

for $t=0$,  \( u(r,0) = 0 \) \hspace{1cm} (0 \leq r \leq R)

for $t>0$,  \( u(R,t) = 0, \ u(0,t)=\text{limited value} \)

where $R$ is tube radius. By solving the momentum equation we obtain the flow rate

$$ Q = 2\pi \int_0^R \left[ \frac{\pi R^4}{8\mu} \left( \frac{\partial p}{\partial x} \right) \left( 1 - 32 \sum_{n=1}^{\infty} \frac{1}{\lambda_n} \exp(-\lambda_n^2 R^2) \right) \right] dr $$

(2)

Here $J_0$ and $J_1$ are the zero-rank and one-rank Bessel functions respectively, and $\lambda_n$ is the eigenvalue of zero-rank Bessel function.

It is clear from Eq. (2) that the flow rate driven by a sudden pressure is directly proportional to the pressure gradient along the tube.

2.3 Mass distribution results

For the situation shown in Fig. 3, the ratio of the mass flows to both sides is

$$ \frac{M_1}{M_2} = \frac{\nabla P_1}{\nabla P_2} \cdot \frac{L_2}{L_1} \cdot \frac{P-P_1-\Delta p_1}{P-P_2-\Delta p_2} \quad (3) $$

where $\Delta p_{1,2}$ is the friction pressure drop.
For laminar flow in a straight tube, the friction pressure drop is

$$\Delta p = f \frac{l}{d} \rho \frac{U^2}{2} = \frac{32 \rho v}{d^2} lU$$  \hspace{1cm} (4)$$

where $f$ is the frictional factor, $d$ is the tube diameter, $U$ is the averaged velocity, $v$ is the kinematic viscosity.

After a bubble separates the fluid in the tube into two parts, the momentum conservation law can be expressed as:

$$\frac{\pi}{4} d^2 l_1 U_1 = \frac{\pi}{4} d^2 l_2 U_2$$  \hspace{1cm} (5)$$

Combining equations (4) and (5), we get

$$\Delta p_1 = \Delta p_2$$  \hspace{1cm} (6)$$

Therefore, if there is no or little pressure difference between inlet and outlet, we have

$$\frac{M_1}{M_2} = \frac{L_2}{L_1}$$  \hspace{1cm} (7)$$

Eq. (7) can be used to predict the cycle mass flux for a given micropump.

If the pressure difference can not be ignored and the inlet pressure is constant $P_0$, there is

$$\frac{M_1}{M_2} = \frac{L_2}{L_1} \frac{P - P_1 - \Delta p_1}{P - P_2 - \Delta p_2} = \frac{P' - P_1}{C_1}$$  \hspace{1cm} (8)$$

$$\frac{M_1}{M} = \frac{M_1}{M_1 + M_2} = \frac{P' - P_1}{C_1 + P' - P_1}$$  \hspace{1cm} (9)$$

where $P' = P - \Delta p$, $C_1 = \frac{L_2}{L_1} (P - P_0 - \Delta p)$.

Eq. (9) can be used only to predict the trend of mass flow versus pump pressure because $P$ and $\Delta h$ are unknown. For more accurate prediction, the unknown parameters must be determined by experiments.

3. Experiments

Experiments were done to verify the theoretical analysis. The experimental system is shown in Fig. 4. The stainless steel microtubes used as micropump bodies were 200 µm and 300 µm in inner diameter. The working fluid was distilled water. DC currents were passed through and switched among the subsections to heat and evaporate the water. The dimensions and subsections of the microtubes are shown in Table 1.

![Fig. 4 The experimental system](image)

<p>| Table 1 Dimensions and subsections of the two microtubes used in the experiments |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|</p>
<table>
<thead>
<tr>
<th>Inner diameter</th>
<th>Outer diameter</th>
<th>Length of tube</th>
<th>Subsections number</th>
<th>Subsection length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube 1</td>
<td>200µm</td>
<td>300µm</td>
<td>30 cm</td>
<td>5</td>
</tr>
<tr>
<td>Tube 2</td>
<td>300µm</td>
<td>400µm</td>
<td>30 cm</td>
<td>7</td>
</tr>
</tbody>
</table>

The maximum flow rate reached 33 µl/min, and the maximum pump pressure was over 20 kPa when the inner diameter was 200 µm\(^8\).

The average volume flow per cycle was also measured. The results are shown in Tab. 2. The differences between experimental and theoretical values are less than 10%, which represents good agreement.
Tab. 2 Volume flow per cycle

<table>
<thead>
<tr>
<th></th>
<th>Experimental value</th>
<th>Theoretical value</th>
<th>Relative Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube 1</td>
<td>2.1µl</td>
<td>2.12µl</td>
<td>&lt;5%</td>
</tr>
<tr>
<td>Tube 2</td>
<td>5.6µl</td>
<td>6.08µl</td>
<td>&lt;10%</td>
</tr>
</tbody>
</table>

Fig. 5 shows the experimental results of pumping pressure versus flow rate, which are of the same trends as the fractional function (as Eq. 9 indicated for the relation of pressure drop to mass flow) shown in Fig. 6.

4. Conclusion

The pumping effect of traveling phase transition in a microtube is modeled and tested. A physical model is established and analyzed. The mass flow to a given micropump is predicted, which is then proved to be in good agreement with experimental results. Analysis based on the model shows that the high instantaneous pressure during evaporation is the main force that drives the micropump.

Acknowledgement

The present work was supported by the Major State Basic Research Development Program of China (999033106) and the National Natural Science Foundation of China (599555550-2).

Reference