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Multi-dimensional effect on optimal network structure for fluid distribution

Xiong-Bin Liu [a], Qun Chen [a,b,*], Moran Wang [c,**], Ning Pan [b], Zeng-Yuan Guo [a]

[a] Key Laboratory for Thermal Science and Power Engineering of Ministry of Education, Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China
[b] Department of Biological & Agricultural Engineering, University of California, Davis, CA 95616, USA
[c] Earth and Environmental Sciences Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

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ABSTRACT

Optimization of a fluid distributing tube network plays a critical role in the operation efficiency and energy conservation of a cooling system. In this paper, we analyze the effect of multi-dimension on the structure of a fluid distributing tube network for cooling heat-generating, and then seek the optimal fluid structure with minimal pressure drop for a given total volume of the tube network. The theoretical results show that the pressure drop of a laminar flow in the tube network reaches the minimum when the cubic value of the parent tube diameter equals to the sum of those of the daughter tube diameter, consistent with Murray's law. Furthermore, it is advantageous to have a higher dimension structure in network arrangement when the number of heat generation units increases, i.e., network performance improves by adopting a two-dimensional and even three-dimensional format as the number of units grows.

1. Introduction

As various systems become more sophisticated and densely integrated, the need for heat dissipation rate of the devices has been rising so quickly to such a significantly high level that simple cooling mode is no longer sufficient, and more complex and effective cooling mechanisms become urgently required. Thus, study of cooling system with fluid distributing network has attracted more and more attention, especially in electronic, chemical, and energy fields [1–3]. This kind of cooling system can be simply described as a network of tubes with cooling fluid, arranged in a device consisting of n heat generation units/elements, while both the heat generation rate and the cooling fluid flow rate, $\dot{G}_0$, in each unit remain equal and constant. Thus, the function of the network is to distribute the cooling fluid with the inlet flow rate, $n\dot{G}_0$, into the n heat generation elements, collect the used cooling fluid after passing through these elements, and finally discharge the fluid from the outlet. In the interest of energy conservation, we would like to optimize the distribution network for the lowest pressure drop between the inlet and outlet at a given flow rate [3,4]. Meanwhile, because the whole volume of the system is related to the total cost, it makes sense to treat the total volume of the cooling fluid distributing network as a constraint.

Duan et al. [5] proposed a method for optimizing the shape of a single fluid distributor based on the variational level set method, in which a relatively smooth flow path is maintained with the minimum flow resistance at a given constant fluid flow rate. Aragon et al. [6] developed a multi-objective genetic algorithm in designing a two-dimensional (2D) and a three-dimensional (3D) microvascular networks embedded in the bio-mimetic self-heating/self-cooling polymeric materials, and investigated the effects of such factors as the network redundancy, template geometry and microchannel diameter on the Pareto-optimal fronts generated by the genetic algorithm. Gosselin and da Silva [7] optimized a rarefied gas distribution network from a source point to a given number of equidistant users with given microscale pipes carrying the fluid. Furthermore, in order to increase the computational efficiency, Saber et al. [8] performed a hydrodynamic analysis of such multiscale networks under isothermal and laminar flow conditions, and then introduced a simple method to quickly select an appropriate numbering-up operation.

Meanwhile, by the analogy between fluid flow and electrical circuits, Zhmoginov and Fisch [9] obtained an optimal exit flux arrangement in networks of intersecting diffusion domains with a special form of thin paths. Also, inspired by the fractal pattern of mammalian circulatory and respiratory systems [10], Chen and Cheng [11] designed a fractal branching channel conformation to cool down electronic chips, exhibiting a greater heat transfer capability yet requiring a lower pumping power, compared to the traditional parallel network. Next, Tondeur and Luo [12–14] exper-

* Corresponding author at: Department of Engineering Mechanics, Tsinghua University, Beijing, 100084, China. Tel.: +86 10 6278 1610.
** Corresponding author.
E-mail addresses: chenqun@tsinghua.edu.cn (Q. Chen), moralwang@gmail.com (M. Wang).

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imentally investigated a typical fluid distributor widely used in catalytic/adsorbent monoliths for homogenously distributing the inlet fluid to a plane surface, where each bigger tube splits into two smaller ones in each subsequent step. They pointed out that the radius ratio for the succeeding tubes in the branching network satisfies the Murray's Law [15], while the viscous dissipation per unit volume in all tubes remains constant so that the overall viscous dissipation in the entire fluid distributor is the lowest.

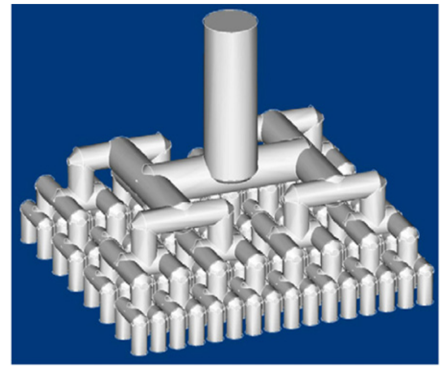
On the other hand, the so-called constructal theory was then introduced for designing the network of conducting path for cooling a heat generation system [16]. Based on the theory, in a system assembled by a number of smaller units, the system properties can be estimated from the performance parameters of the constituents [16]. Furthermore, Gosselin [17,18] discussed the influence of turbulence flow instead of the usual laminar one in a tube on the optimal structure of the flow tube network. Besides, Fan et al. [19] in their own experiments investigated the flow distribution behavior of a plate-type constructal flow distributor, which has one inlet and 16 outlets and was designed according to the constructal theory to achieve a uniform flow distribution with the smallest mechanical energy dissipation and the shortest residence time. However, the validity of the constructal theory was also questioned by other scholars. Ghodoossi [20] found that the supposedly optimized results from the constructal theory were in fact not optimal with the increasing constructal steps, i.e. the increasing structure complexity. Likewise, Wu et al. [21] obtained a result with performance better than the one based on the constructal theory [16] and thus challenged the assumption that a system is automatically optimal if consisting of optimal constituents. Overall, in the above references, based on the constructal theory, the optimized networks usually have hierarchal fractal networks as shown in Fig. 1 [16–18], but the influences of the branch number and the system dimension on the performance of such tube network have not been explicitly discussed in those reports.

In this paper, several cooling fluid distributing networks composed of many heat generation units are investigated. The pressure drops of the distributing networks in one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D) cases are calculated, the relationship between the dimension of the optimal fluid distributing network and the number of heat generation elements is discussed, and finally with the constraint of a constant total volume, some new design approaches for optimization in network design with minimal pressure drop and the major factors involved are developed.

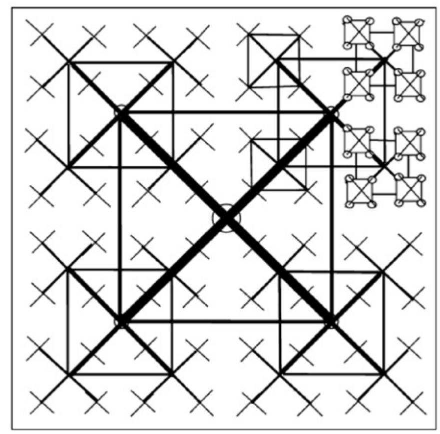
2. 1D structure optimization

Consider a cubic heat generation element with length, width and height of x_0, y_0 and z_0, respectively, where $x_0 < y_0 < z_0$. $n = pqr$ such heat units form a practical heat generator, with p, q, and r units in x, y, and z direction, respectively. Thus, the total length, width and height of the heat generator are px_0, qy_0 and rz_0. The cooling fluid flow rates $\dot{G}_0$ in each heat generation element are the same, so the total cooling fluid flow rate in the entire heat generator is $n\dot{G}_0$. Because the main objective of this paper is to study the cooling fluid distributing process, for brevity the details in the heat transfer process between the cooling fluid and each heat unit is not included.

Ideally, the cooling fluid needs to be distributed homogenously into the n units and collected by a fluid distributing tube network similar to Fig. 1. For a single heat unit clearly, neither the fluid distributing network nor its optimization is existent. In the case that the heat generator is composed of two or more elements however, we need a network to distributing the fluid to cool these elements uniformly. Because the electronic elements and the cabinets are usually the source for expenditure, the total volume V_{all} of the



(a) The fluid distributor with 128 flow outlets by 7 two-bifurcates [13]



(b) The overlook sketch of fluid distributor with four-bifurcates [12]

Fig. 1. Sketches of some fluid distributors. (a) The fluid distributor with 128 flow outlets by 7 two-bifurcates [13]; (b) the overlook sketch of fluid distributor with four-bifurcates [12].

cooling fluid distributing network is taken as the constraint for optimization. Note here V_{all} is not the total volume of all the heat units $V = nx_0y_0z_0$, but the total volume of all the tubes in the distributing network, derived from the diameter and length of each tube.

On condition that at least two or more heat generation elements are in a heat generator, the simplest method to design the cooling fluid distributing network for the system is that each element is connected by two tubes side by side, one distributing and the other collecting the fluid. Fig. 2 shows such a distributing tube network with the heat units arranged in x direction. For clarity, the two heat generation elements at a given gap in between are shown in the left of Fig. 2, while in the right, the solid lines stand for the fluid distributing/collecting tubes and dashed lines the heat exchange tubes.

Considering the fluid distributing network with all straight tubes in Fig. 2, the total volume of the tubes V_{all}, except for the heat exchange tubes, is readily calculated as

$$V_{all} = \frac{(n-1)\pi D_x^2}{2} x_0, \tag{1}$$

where D_x is the diameter of the tube in x direction.

Assuming laminar flows are fully developed in all tubes and the pressure drop caused by the heat exchange tube is neglected, the total pressure drop of the cooling fluid flowing from the inlet A to the outlet B is expressed as

$$\Delta P_1 = \frac{128\mu \sum_{i=1}^{n-1} (i\dot{G}_0)}{\pi D_x^4} x_0, \tag{2}$$

where μ is the fluid viscosity, and $\dot{G}_0$ the flux shown in Fig. 2.

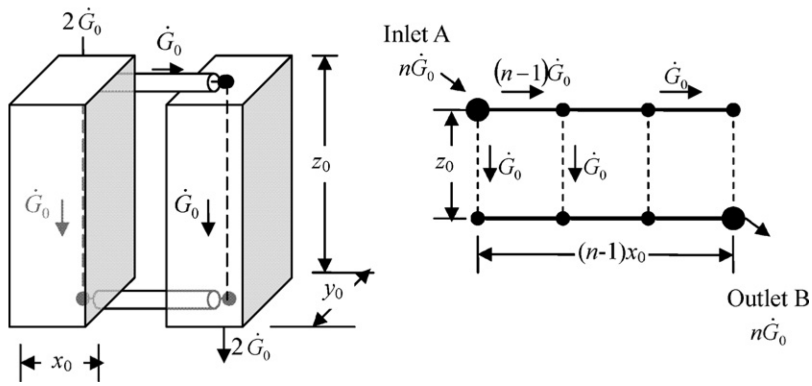


Fig. 2. Arrangement of two or more heat generation elements in x direction.

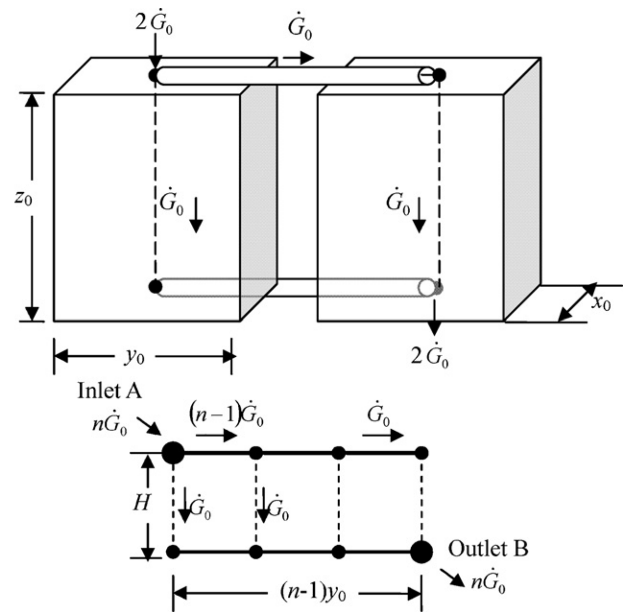


Fig. 3. Arrangement of two or more heat generation elements in y direction.

Substituting Eq. (1) into Eq. (2) yields the total pressure drop of the one-dimensional fluid distributing network:

$$\Delta P_1 = \frac{16\pi\mu(n-1)^3 x_0^3}{V_{all}^2} n\dot{G}_0. \tag{3}$$

Similarly, when the heat generation elements are arranged in y direction alone shown in Fig. 3, the total pressure drop becomes:

$$\Delta P_2 = \frac{16\pi\mu(n-1)^3 y_0^3}{V_{all}^2} n\dot{G}_0. \tag{4}$$

Because the length of the heat generation element in x direction is less than that in y direction, the pressure drop in Fig. 2 is smaller than that in Fig. 3, which is in turn smaller than the one when the heat generation elements are arranged in z direction. Thus, the tube network shown in Fig. 2 is the optimal 1D structure for the cooling fluid distributing network.

3. 2D structure optimization with $p \times q$ heat generation elements

As shown in Fig. 4 for the 2D case, a heat generator is composed of $n = p \times q$ units. The cooling fluid is first uniformly distributed in y direction to each tube along x direction, and then to each heat generation element along the way, termed the "first y next x" approach in Fig. 4. Similarly, the fluid can also be distributed in x direction to each tube along y direction, and then to each element, as the "first x next y" approach in Fig. 4.

For the "first y next x" tube network, assuming the diameter of the tubes along x and y directions as D_x, and D_y, respectively, and again ignoring the influence of the heat exchanging tubes, the

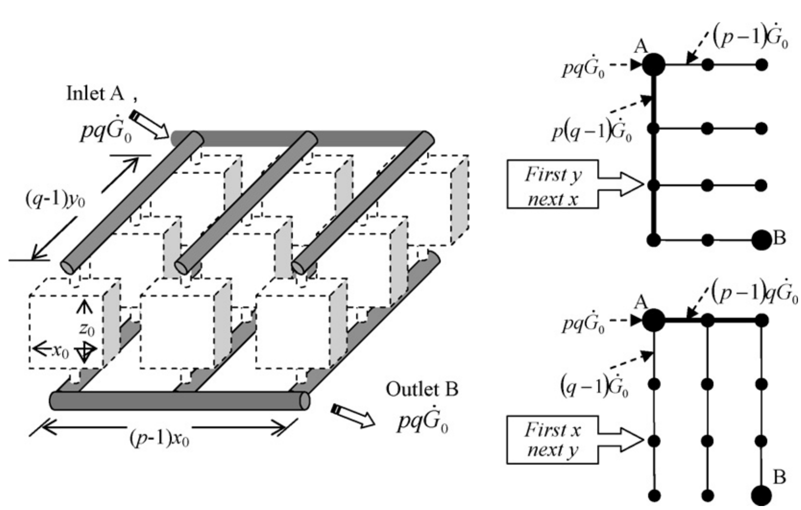


Fig. 4. Two-dimensional tube network arrangement of $p \times q$ heat generation elements.

pressure drop from the inlet A to the outlet B can be expressed as

$$\Delta P_3 = \frac{128\mu\sum_{i=1}^{q-1}(pi\dot{G}_0)}{\pi D_y^4}y_0 + \frac{128\mu\sum_{i=1}^{p-1}(i\dot{G}_0)}{\pi D_x^4}x_0. \tag{5}$$

The total volume of all the tubes, except the heat exchange ones, is

$$V_{all} = \frac{(p-1)q\pi D_x^2}{2}x_0 + \frac{(q-1)\pi D_y^2}{2}y_0. \tag{6}$$

Based on Eqs. (5) and (6), we find that the pressure drop of the "first y next x" tube network reaches the minimum when $D_x/D_y = q^{-1/3}$, i.e., the optimal diameter ratio of the adjacent two tubes is determined by the number of the daughter tubes q connected to the parent tube. A smaller q leads to a greater ratio of D_x/D_y, as shown in Eq. (8). Here, the lowest pressure drop between the inlet A and the outlet B is

$$\Delta P_{3,\min} = \frac{16\pi\mu[(p-1)q^{1/3}x_0 + (q-1)y_0]^3}{V_{all}^2}pq\dot{G}_0. \tag{7}$$

The expression, $D_x/D_y = q^{-1/3}$ is equivalent to

$$D_y^3 = qD_x^3. \tag{8}$$

As Murray's Law [5] indicated, the cube of the radius of a parent vessel equals to the sum of the cubic radius of the daughter vessels in an organism. In the fluid distributing tube network studied in this paper, due to mass conservation, the flow rate in each parent tube equals to the total flow rates in q daughter tubes. As expressed in Eq. (8), the fluid flow rates in the parent and the daughter tubes are proportional to the cubic values of their corresponding diameters. That is to say, Eq. (8) is equivalent to Murray's law for the aforementioned tube network.

Similarly, for the "first x next y" network arrangement, the pressure drop is lowest when $D_y/D_x = p^{-1/3}$, i.e. a larger number p will lead to a greater diameter D_y of the daughter distributing tube than the diameter D_x of the parent one. The lowest pressure drop of the "first x next y" tube network between the inlet A and the outlet B now is

$$\Delta P_{4,\min} = \frac{16\pi\mu[(p-1)x_0 + (q-1)p^{1/3}y_0]^3}{V_{all}^2}pq\dot{G}_0. \tag{9}$$

4. Transition of optimal network from 1D to 2D

Eqs. (7) and (9) give the lowest pressure drop in the fluid distributing network for $p \times q$ heat generation elements, arranged as in Fig. 4. Alternatively, the network can also be constructed as Fig. 5, whose pressure drop can then be calculated approximately by using

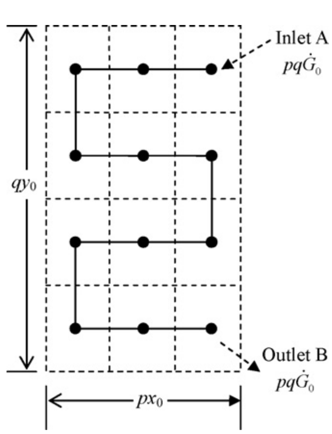


Fig. 5. The snakelike distributing tube network for $p \times q$ heat generation elements.

the pressure drop formula Eq. (3) in one-dimensional arrangement in Fig. 2. The amount of the elements, $n = p \times q$, is substituted into Eq. (3) to calculate the pressure drop for the snakelike structure in Fig. 5. Thus, for a heat generator consisted of $p \times q$ elements, three possible arrangements exist, i.e. the snakelike one-dimensional one, the "first y next x" and the "first x next y" two-dimensional arrays, respectively. The following is the comparison of the pressure drop in these three cases.

From Eqs. (3) and (7), it is clear that the pressure drop of the one-dimensional network is less than that of "first y next x" tube network if

$$p \le \frac{y_0}{x_0} + \frac{y_0/x_0 - 1}{q^{2/3} + q^{1/3}}. \tag{10}$$

When $q \gg 1$ or the ratio y_0/x_0 approximates to 1, the condition in Eq. (10) is reduced into

$$\frac{pqx_0}{qy_0} \le 1. \tag{11}$$

Similarly, comparing Eq. (3) and Eq. (9), the pressure drop in the one-dimensional case lower than that in "first x next y" network if

$$p \le \left(\frac{y_0}{x_0}\right)^{3/2}. \tag{12}$$

From Eq. (7) and Eq. (9), the condition that "first y next x" tube network is better than "first x next y" tube network is

$$\frac{x_0}{y_0} \le \frac{q^{2/3} + q^{1/3} + 1}{p^{2/3} + p^{1/3} + 1}. \tag{13}$$

When both the ratio, x_0/y_0, and the value q of the elements in y direction are given, Eq. (13) can be simplified into a quadratic inequality with an unknown variable $p^{1/3}$ with only one non-trivial solution, i.e.,

$$p_c = \left[\sqrt{\frac{y_0}{x_0}q^{2/3} + \frac{y_0}{x_0}q^{1/3} + \frac{y_0}{x_0} - \frac{3}{4}} - \frac{1}{2}\right]^3. \tag{14}$$

Thus, of the two 2D arrangements, the pressure drop of "first y next x" network is lower when $p < p_c$, and the "first x next y" tube network is better when $p > p_c$. As shown in Eq. (14), the larger the ratio y_0/x_0, the wider the range $[0, p_c]$ for p, and thus a lower pressure drop for the "first y next x" network.

If the number q of the elements in y direction is large enough so that $(y_0/x_0)q^{2/3}$ becomes much greater than any other items in Eq. (14), then, Eq. (14) is simplified into

$$p_c = \left(\frac{y_0}{x_0}\right)^{3/2} q. \tag{15}$$

Eq. (15) can be used as an approximate criterion to compare the performance between the "first y next x" and the "first x next y" networks. More importantly, Eq. (15) connects the macro-size of the heat generator $(px_0)/(qy_0)$ with the element size x_0/y_0, clearly useful for design and other applications in engineering. For instance, for given element dimension x_0/y_0 and p_c, by adjusting q we can design a network in either 2D format with lower pressure drop.

In summary and on assumption $q \gg 1$, the relationship and comparison of the pressure drops for the three network arrangements are concluded in Table 1.

As shown in Table 1, for a given size of the heat generator and heat element, there exists the following relations in terms of the pressure drops: from the first and second rows, if 1D is not the best of the three, then 2DYX is always a better design than 2DXY; if 1D is the best as in rows 3 and 4, there is no definite relation between 2DYX and 2DXY.

From Eq. (11), we can further conclude that: if *the characteristic length* pqx_0 of a one-dimensional network is smaller than the

Table 1
Pressure drops for three network arrangements: one-dimensional (1D), two-dimensional “first *y* next *x*” (2DYX) and “first *x* next *y*” (2DXY).

Exact range for p	Approximate range for p	Comparison of pressure drop
$1 < p < \frac{y_0}{x_0} + \frac{y_0/x_0-1}{q^{2/3}+q^{1/3}}$	$1 < p < \frac{y_0}{x_0}$	1D < 2DYX < 2DXY
$\frac{y_0}{x_0} + \frac{y_0/x_0-1}{q^{2/3}+q^{1/3}} < p < \left(\frac{y_0}{x_0}\right)^{3/2}$	$\frac{y_0}{x_0} < p < \left(\frac{y_0}{x_0}\right)^{3/2}$	2DYX < 1D < 2DXY
$\left(\frac{y_0}{x_0}\right)^{3/2} < p < \left[\sqrt{\frac{y_0}{x_0}q^{2/3} + \frac{y_0}{x_0}q^{1/3} + \frac{y_0}{x_0} - \frac{3}{4}} - \frac{1}{2}\right]^3$	$\left(\frac{y_0}{x_0}\right)^{3/2} < p < q\left(\frac{y_0}{x_0}\right)^{3/2}$	2DYX < 2DXY < 1D
$\left[\sqrt{\frac{y_0}{x_0}q^{2/3} + \frac{y_0}{x_0}q^{1/3} + \frac{y_0}{x_0} - \frac{3}{4}} - \frac{1}{2}\right]^3 < p$	$q\left(\frac{y_0}{x_0}\right)^{3/2} < p$	2DXY < 2DYX < 1D

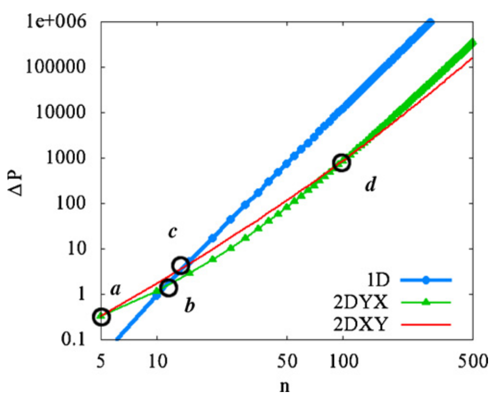


Fig. 6. Pressure drop in different arrangements for given number of elements $n = pq$ ($x_0/y_0 = 0.5$ and $q = 5$).

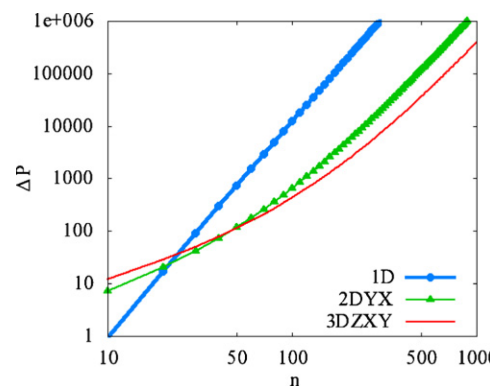


Fig. 7. Pressure drop in different arrangements for given number of elements $n = pqr$ ($x_0/y_0 = 0.5$, $y_0/z_0 = 0.5$, $q = 5$ and $r = 2$).

parent length qy_0 in a 2DYX network, then the 1D network is a better choice, i.e., resulting in a lower pressure drop. Otherwise, the reverse is true. Or in more brief terms, the approximate criterion for judging one- or two-dimensional cooling flow distributing networks can be expressed as *the shorter the characteristic length, the better the network*.

To further elucidate the results from Eq. (10) to Eq. (15), assuming $x_0/y_0 = 0.5$ and $q = 5$, and the total heat generation elements $n = pq$, Fig. 6 and Table 2 present the dimensionless pressure drop for the three cooling flow distributing networks illustrated in Figs. 5 and 6.

In the figure there are four points of intersection by the three curves, i.e. Points *a*, *b*, *c* and *d*. The physical meanings of these points are as follows. Point *a* corresponds to the case $n = 1$, i.e. $p = 1$, where 2DXY = 2DYX; point *b* indicates the conditions for 1D = 2DYX, when $p \le (y_0/x_0) + (y_0/x_0 - 1)/(q^{2/3} + q^{1/3}) \approx 2.2$ and $n = pq \le 11$, as shown in Eq. (10), below which the ID arrangement becomes the best owing to the lowest pressure drop. Point *c* stands for the critical value 1D = 2DXY from Eq. (12) when $p \le (y_0/x_0)^{3/2} = 8^{1/2}$, i.e. $n \le 200^{1/2}$. Once again, 2DXY = 2DYX at point *d* as defined in Eq. (14) when $p \le \left[\sqrt{(y_0/x_0)q^{2/3} + (y_0/x_0)q^{1/3} + (y_0/x_0) - (3/4)} - (1/2)\right]^3 \approx 20.6$, i.e. $n = pq \le 103$. Overall, a complete criterion can be formed by the envelope connecting all the segments located between the four crossing points, for such tube network design of different size *n* as

Table 2
Dimensionless pressure drop for various tube networks at given sizes n ($x_0/y_0 = 0.5$ and $q = 5$).

p	$n = pq$	ΔP for 1D network	ΔP for 2DYX network	ΔP for 2DYX network
1	5	4.00×10^{-2}	3.20×10^{-1}	3.20×10^{-1}
2	10	9.11×10^{-1}	1.14	1.70
3	15	5.15	2.79	4.65
20	100	1.21×10^{4}	8.30×10^{2}	8.44×10^{2}
21	105	1.48×10^{4}	9.86×10^{2}	9.77×10^{2}
100	500	7.77×10^{6}	3.48×10^{5}	1.58×10^{5}

indicated in Table 2. In addition, it is important to note that Fig. 7 is a logarithmic plot. Thus, pressure drops vary more markedly than appeared in the figures.

5. 3D structure optimization with $p \times q \times r$ heat generation elements

The third option to design the cooling fluid distributing tube network using given heat generation elements is that the cooling fluid is homogeneously distributed into the heat generation units in three steps. For the heat generator composed of $p \times q \times r$ heat units in *x*, *y* and *z* directions, the fluid can be first distributed from the tube in *z* direction to the inlet of the daughter tube network of $p \times q$ at a given *x*–*y* plane, and then distributed by the 2DXY or 2DYX methods, leading to six 3D arrangements with different combination of *X*, *Y*, and *Z*.

For instance in the arrangement 3DZXY, the diameters of tubes in *x*, *y* and *z* directions are D_x, D_y and D_z, respectively, and the pressure drop of the tube network is expressed as

$$\Delta P_5 = \frac{128\mu\sum_{i=1}^{p-1}(qi\dot{G}_0)}{\pi D_x^4}x_0 + \frac{128\mu\sum_{i=1}^{q-1}(i\dot{G}_0)}{\pi D_y^4}y_0 + \frac{128\mu\sum_{i=1}^{r-1}(pqi\dot{G}_0)}{\pi D_z^4}z_0. \quad (16)$$

The total volume of all the tubes, except the heat exchange ones, is again

$$V_{all} = \frac{(p-1)R\pi D_x^2}{2}x_0 + \frac{p(q-1)r\pi D_y^2}{2}y_0 + \frac{(r-1)\pi D_z^2}{2}z_0. \quad (17)$$

Based on Eqs. (16) and (17), the pressure drop in this 3DZXY tube network reaches the minimum when

$$D_y/D_x = p^{-1/3} \quad \text{and} \quad D_x/D_z = r^{-1/3}. \quad (18)$$

Table 3
Dimensionless pressure drop for various tube networks at given sizes $n = pqr$ ($x_0/y_0 = 0.5$, $y_0/z_0 = 0.5$, $q = 5$ and $r = 2$).

p	$n = pqr$	ΔP for 1D network	ΔP for 2DYX network	ΔP for 3DZXY network
1	10	9.11×10^{-1}	7.29	1.20×10^{1}
2	20	1.71×10^{1}	2.05×10^{1}	2.85×10^{1}
3	30	9.15×10^{1}	4.16×10^{1}	5.03×10^{1}
4	40	2.97×10^{2}	7.32×10^{1}	7.83×10^{1}
5	50	7.35×10^{2}	1.18×10^{2}	1.13×10^{2}
100	1000	1.25×10^{8}	1.55×10^{6}	3.89×10^{5}

Here, the lowest pressure drop is

$$\Delta P_{5,\min} = \frac{16\pi\mu\left[(p-1)r^{1/3}x_0 + (q-1)p^{1/3}r^{1/3}y_0 + (r-1)z_0\right]^3}{V_{all}^2} \times pqr\dot{G}_0. \quad (19)$$

Besides such six three-dimensional arrangements, the $p \times q \times r$ heat generation elements can also be arranged in a one-dimensional format shown in Fig. 2, or three two-dimensional arrangements with p rows and qr columns, pq rows and r columns, or pr rows and q columns, respectively. Each two-dimensional arrangement has again two conformations, i.e. 2DXY and 2DYX. For brevity and comparison purpose, we analyze in this paper only the 1D arrangement in Fig. 2, the 2DYX arrangement with p rows and qr columns in Fig. 5, and the 3DZXY.

Fig. 7 and Table 3 give the results at $n = pqr$, $q = 5$ and $r = 2$ for the three cases. The figure shows that with increasing number of the heat generation elements, the optimal arrangement transcends from 1D at $n < 20$, to 2D at $20 < n < 50$, and finally to 3D at $n > 50$. That is, when the number of heat units increases, the arrangement at higher dimension offers a better performance.

6. Conclusions

Structure optimization of fluid distributing tube network for cooling systems has been investigated in this paper to emphasize the effect of multi-dimensional conformation. The concerned system consists of one single inlet and outlet and a fluid tube network in between, and can be in one-, two- or three-dimensional arrangement. The one-dimensional network will exhibit better performance, i.e., with lowest pressure drop if the number of the element $n = p \leq (y_0/x_0)^{3/2}$, where (y_0/x_0) is the prescribed element aspect ratio. Beyond this n value, two-dimensional arrangement becomes more favorable. As the total number of elements n increases further, a three-dimensional network will provide better performance. In other words, as the number of heat generation elements (the total system size) n increases, the optimal structure of fluid distributing tube network for the lowest pressure drop will transit from one-dimensional to two-dimensional, and finally to three-dimensional structures.

Most importantly, the present contribution points out that in a fluid distributing network the optimal branch level (or dimension) of a fluid distributing network should be carefully chosen based on the problem scale, and more branch layers do not always lead to better performance.

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