Gas permeability calculation of tight rocks based on laboratory measurements with non-ideal gas slippage and poroelastic effects considered

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A R T I C L E   I N F O

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Tight rocks
Poroelastic deformation
Gas slippage
Real gas effect

A B S T R A C T

Permeability measurements on low-permeable rocks in the laboratory require much higher pressure gradients than those in real reservoirs to produce detectable flow rates in finite laboratory time. This may result in a high effective stress gradient that can cause non-uniform deformation of the pore system. To better understand the measured laboratory data, a theoretical model has been developed for calculating gas permeability of tight rock from laboratory measurements, which couples the effect of poroelastic deformation with the gas non-ideality and slippage effects. The proposed characteristic pressure model considers the poroelastic deformation and the real gas effects in the permeability calculation, which improves the accuracy of calculated permeability from laboratory measurements of tight rocks under large pressure gradients. The new model is validated by independent multiscale simulations, in which the poroelastic deformation and slippage effects are captured on the pore scale while the real gas behavior is captured on the core scale. The numerical results also indicate that the poroelastic deformation mainly affects the high-pressure region while the variation of gas properties dominates the low-pressure region. The new model is then applied to the calculation of gas permeability based on the laboratory measurements on coal and shale samples with non-ideal gas slippage and poroelastic effects considered. The poroelastic deformation and the real gas effect can be important as well as the slippage effect and the calculated apparent permeability will be overestimated if these two effects are neglected.

1. Introduction

Gas flow in tight rocks is involved in many important applications such as carbon dioxide sequestration, nuclear waste disposal and unconventional gas exploitation. Accurate permeability measurements on low-permeable rocks are essential to make predictions and optimizations in these applications. A popular method is the unsteady-state measurement (e.g. the transient pulse decay). This method is faster than the conventional steady-state method and has been developed to take into account the gas compressibility and slippage, which are essential in gas permeability measurement. However, recent experiments have shown that the unsteady-state method consistently overestimates permeability values. Rushing et al. found that when the steady-state method is corrected for gas slippage, the results agree well with liquid permeability coefficients, but the unsteady-state method overestimates the permeability by more than 80% when it is less than 0.01 mD. Similarly, Carles et al. reported that the permeability measured by the unsteady-state method is about two times the permeability measured by the steady-state method or the GRI method (i.e. pulse pressure testing of core chips or drill cuttings).

These results suggest that the steady-state method may be more reliable than the unsteady-state method. However, using the steady-state method to measure the permeability of tight rocks is also challenging. Because of the low permeability coefficient, measuring a low (geological) pressure gradient in an intolerable time to reach steady state. Besides, a low flow rate is difficult to measure accurately and will induce significant error in permeability calculations. Thus, the pressure gradient needs to be greatly elevated to generate larger flow rates and reduce the duration of the experiments, which has been adopted in many studies. A large pressure gradient may result in two important effects besides the traditional gas slippage effect: the poroelastic deformation and the real (non-ideal) gas behavior. These effects need to be, but have not yet been, fully considered in permeability calculations.


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The porous microstructure may deform elastically under the influence of effective stress.\textsuperscript{10,13,14} With a given downstream pressure, a larger pressure gradient means a higher upstream pressure. As the confining pressure is constant during a measurement, a higher pore pressure at the upstream reduces the effective stress, thus the pore throats widen and the local intrinsic permeability is larger compared to the downstream.\textsuperscript{15,16} When a high pressure gradient is applied, the non-uniform deformation becomes significant. However, the impact of this effect on permeability calculation has barely been investigated in previous works.\textsuperscript{8–11} 

Besides, at high upstream pressure, the properties of a real gas may deviate from those of the ideal gas, including a non-linear relationship between gas density and pressure (equation of state) and a pressure-dependent dynamic viscosity.\textsuperscript{17–19} This phenomenon leads to a correction in the Knudsen number calculation, which is much underestimated by the ideal gas assumption at high gas pressures.\textsuperscript{20} Besides, the real gas properties affect the pressure profile along the core, which complicates the calculation of permeability coefficients. To model the real gas effect, Ghani zadah et al.\textsuperscript{11} replaced the ideal gas viscosity by the real gas viscosity at the mean gas pressure. Rushing et al.\textsuperscript{5} further assumed a constant compressibility factor (Z) and selected its value at mean pressure. Both models used constant properties at the mean pressure to describe real gas behavior over the entire pressure range, which may result in over- or under-estimations due to the local variation of gas properties.

Both the poroelastic deformation and the real gas effects are coupled with the slippage effect because the pore size is affected by deformation while the mean free path is dependent on gas density and viscosity. In this work, we attempt to theoretically take into account all three effects in a unified model. We first present the derivation of the new model, then validate it through independent numerical simulations and finally apply this model to laboratory data for a more accurate interpretation.

2. Theoretical model

We consider a steady-state gas flow through a core sample with length $L$ in Fig. 1. The upstream pressure, downstream pressure and confining pressure are $p_1$, $p_2$ and $p_c$, respectively. The heterogeneity perpendicular to the flow direction is neglected, so that the porous flow can be simplified to a one-dimensional problem. The following derivation is a further development of our recent work,\textsuperscript{21} incorporating the poroelastic deformation with the high-Kn and real gas effects.

For a high-Kn flow in porous media, the traditional Darcy’s theory is no longer valid. Klinkenberg introduced the pressure-dependent permeability into Darcy’s law,\textsuperscript{12} which has been widely used and physically explained in later studies.\textsuperscript{22–25} The mass flux is related to the pressure gradient as:

$$Q_m = -\rho(p) \frac{K(p)}{\mu(p)} \frac{dp}{dx} = -\frac{K(p)}{\nu(p)} \frac{dp}{dx},$$

where $\rho$ is the density, $K$ the apparent permeability coefficient, $\mu$ the dynamic viscosity and $\nu(p) = \mu(p)/\rho(p)$ the kinematic viscosity. After separating variables and integrating from upstream to downstream, we have:

$$\int_0^L Q_m dx = -\int_{p_1}^{p_2} \frac{K(p)}{\nu(p)} dp.$$  \hfill (2)

For a steady-state flow, the mass flux is constant over the entire length of the sample:

$$Q_m = \frac{1}{L} \int_{p_1}^{p_2} \frac{K(p)}{\nu(p)} dp.$$  \hfill (3)

In the slip-flow regime, the apparent permeability coefficient is proportional to the Knudsen number ($Kn$)\textsuperscript{26}:

$$K = K_{\infty}[1 + bKn],$$  \hfill (4)

where $K_{\infty}$ is the intrinsic permeability coefficient and $b$ is the Kn-based slip factor. The Knudsen number is defined as\textsuperscript{27}:

$$Kn = \frac{\lambda}{L}. $$  \hfill (5)

Here $\lambda$ is the mean free path of gas molecules and $L$ the characteristic length of pores.\textsuperscript{24} The mean free path of the gas molecules is calculated by the hard-sphere model\textsuperscript{28}:

$$\lambda = \frac{\mu(p)}{\rho(p)} \sqrt{\frac{\pi}{2 R_g T}} = \nu(p) \sqrt{\frac{\pi}{2 R_g T}},$$  \hfill (6)

where $R_g$ is the specific gas constant (the universal gas constant divided by the molar mass), $T$ the temperature and $\nu(p) = \mu(p)/\rho(p)$ the kinematic viscosity. Combination of Eqs. (4)–(6) leads to:

$$K(p) = K_{\infty}[1 + \frac{b}{L} \nu(p)] \sqrt{\frac{\pi}{2 R_g T}}.$$  \hfill (7)

Because of the poroelastic deformation, the intrinsic permeability $K_{\infty}$ and characteristic length $l$ may vary with the pressure conditions. Generally, the confining pressure is constant in a measurement, thus the intrinsic permeability $K_{\infty}$ and the characteristic length $l$ depend on the pore pressure only:

$$K_{\infty}(p) = \gamma_k(p) K_{\infty,0},$$  \hfill (8)

$$l(p) = \gamma_l(p) l_c.$$  \hfill (9)

Here $\gamma_k$ and $\gamma_l$ are dimensionless parameters representing the variation of permeability and characteristic length with pore pressure.

Combination of Eqs. (7)–(9) leads to an expression for the apparent permeability $K$ at the pore pressure $p$:

$$K(p) = K_{\infty,0} \gamma_k(p)[1 + \frac{b \nu(p)}{l_c \gamma_l(p)} \sqrt{\frac{\pi}{2 R_g T}}],$$  \hfill (10)

Substitution of Eq. (10) into Eq. (3) gives:

$$Q_m = \frac{K_{\infty,0}}{L} \int_{p_1}^{p_2} \frac{\gamma_k(p)}{\nu(p)} dp + \frac{b}{l_c} \frac{\sqrt{\pi}}{2 R_g T} \int_{p_1}^{p_2} \gamma_l(p) \nu(p) dp.$$  \hfill (11)

Generally, a specific pressure value $p_{\text{bar}}$ always exists to satisfy the following equation:

$$\frac{\nu(p_{\text{bar}})}{\nu(p)} = \frac{\int_{p_1}^{p_2} \gamma_k(p) \nu(p) dp}{\int_{p_1}^{p_2} \gamma_l(p) \nu(p) dp}.$$  \hfill (12)

Since $\gamma_l$ increases while $\nu$ decreases with pressure, only one $p_{\text{bar}}$ exists. Then Eq. (11) becomes:

$$Q_m = \frac{K_{\infty,0}}{L} [1 + \frac{b}{l_c} \frac{\sqrt{\pi}}{2 R_g T} \gamma_l(p_{\text{bar}}) \int_{p_1}^{p_2} \gamma_k(p) \nu(p) dp].$$  \hfill (13)
According to Eq. (10), the apparent permeability at $p_{\text{char}}$ is:

$$K(p_{\text{char}}) = K_m \bar{\gamma}_l(p_{\text{char}})(1 + \frac{h\varphi(p_{\text{char}})}{l_0\gamma_l(p_{\text{char}})} \sqrt{\frac{\pi}{2K_l T}}).$$

(14)

Combining Eq. (13) and Eq. (14), we finally have:

$$K(p_{\text{char}}) = \frac{Q_m K_l(p_{\text{char}})}{f_{p_1} f_{p_2} \frac{v(p)}{v(p)}} dp.$$  

(15)

With Eq. (15), the apparent permeability is determined using the measured mass flux $Q_m$. Since the apparent permeability is calculated at $p_{\text{char}}$, we define this pressure $p_{\text{char}}$ as the characteristic pressure of the porous flow system. This is different from the previous methods, in which the apparent permeability is calculated at the mean pressure. 6,12

To obtain the value of characteristic pressure $p_{\text{char}}$, the expressions for $\bar{\gamma}_K$ and $\bar{\gamma}_l$ are needed. There are several empirical expressions for $\bar{\gamma}_K$ describing how the intrinsic permeability varies with effective stress,29–31 but $\bar{\gamma}_l$ remains unknown in most cases. If the ratio between $\bar{\gamma}_K$ and $\bar{\gamma}_l$ does not change too significantly, we can take the following approximation to eliminate $\gamma_l$ in Eq. (12):

$$\frac{\bar{\gamma}_K(p_{\text{char}})}{\bar{\gamma}_l(p_{\text{char}})} = \frac{1}{p_1 - p_2} \int_{p_1}^{p_2} \frac{\gamma_l(p)}{v(p)} dp,$$

(16)

which is strictly valid when $\gamma_K/\gamma_l = \text{constant}$. In the numerical validation, we will show that the error of this approximation is only 1.6% in a case that the intrinsic permeability at upstream is 2.74 times as large as the value at downstream and $\gamma_K/\gamma_l$ ranges from 1 to 1.96. By substituting Eq. (16) into (15), the apparent permeability at the characteristic pressure $p_{\text{char}}$ is calculated as:

$$K(p_{\text{char}}) = \frac{Q_m K_l(p_{\text{char}})}{p_1 - p_2}.$$

(17)

In summary, the calculation of the apparent permeability requires two steps. The first step is to determine the characteristic pressure, which is the pressure $p_{\text{char}}$ that satisfies Eq. (16), i.e., characteristic pressure is reached when $\bar{\gamma}_K/\bar{\gamma}_l$ equals its arithmetic mean value between $p_1$ and $p_2$. Since the $\bar{\gamma}_K$ represents how the intrinsic permeability varies with pressure while the calculation of permeability also needs $\gamma_l$, an iteration process may be needed. The second step is to calculate the apparent permeability at the characteristic pressure with Eq. (17). Eq. (17) has the same form as the classical Darcy’s law for incompressible fluids in non-deformable rock, which means physically the deformable rock is equivalent to a rigid structure and the real gas to an incompressible fluid using the rock and gas properties at $p_{\text{char}}$. Besides, this characteristic pressure model will recover the well-known Klinkenberg model Eq. (35) once the porous structure is non-deformable and the gas is ideal, in which case the characteristic pressure equals the mean pressure $\bar{p} = (p_1 + p_2)/2$.

3. Numerical validations

Numerical simulations are employed to validate the characteristic pressure model. The simulations serve as substitutes of real experiments, whose mass flow rates are used as inputs for the model. Although the characteristic pressure model is built for porous media, in the validation the structure is simplified as a slit so that a theoretical value of the permeability can be computed. We consider the gas flow between two elastic plates, the distance between which is dependent on the gas pressure (Fig. 2). The length of the slit is in centimeters to represent the core size, while the height is in the scale of 100 nm, representing the pore size of tight rocks.

Because of the poroelastic effect, the slit height increases with the pore pressure:

$$l(p) = \gamma_l(p) l_0.$$

(18)

The intrinsic permeability of the two-plate structure is:
\[ p_{\text{new}}(i) = \frac{p(i - 1)C(i - 1) + p(i + 1)C(i)}{C(i - 1) + C(i)}, \quad i = 2, 3, \ldots, n. \]  
where \( C(i) \) is the conductivity, defined by the mass flow rate divided the pressure gradient:

\[ C(i) = \frac{M}{\rho v}. \]  

Step (4): Check the deviation between the new pressure distribution and the old one. Go back to Step (1) if the deviation is greater than the tolerance. After the iteration, the mass flow rate \( M \) will be constant and the mass flux is:

\[ Q_{\text{m}} = \frac{M}{H}. \]  

The lattice Boltzmann method (LBM) is employed for the flow simulation in each element. LBM is based on the Boltzmann equation and it has a solid physical foundation to describe high-Kn gas flows. It also has good accuracy and efficiency for pore-scale simulations.

LBM solves the evolution of a discrete distribution function:

\[ f_i(x + c_i \Delta t, t + \Delta t) - f_i(x, t) = \Omega_i, \quad i = 0, 1, \ldots, N - 1 \]  
where \( f_i(x, t) \) is the discrete distribution function, \( c_i \) the lattice speed, \( \Omega_i \) the discrete collision operator and \( N \) the number of the discrete velocities. For good numerical accuracy, we adopt the D3Q19 (3-dimension 19-speed) lattice model and the multiple relaxation time model (MRT). In the MRT model, the discrete collision operator is expressed as:

\[ \Omega_i = -\sum_j (\mathbf{M}^{-1} \mathbf{S})_{ij} (f_j - f_j^{\text{eq}}). \]  

Here \( \mathbf{M} \) is a 19 \times 19 constant matrix; \( \mathbf{S} \) is a diagonal matrix:

\[ \mathbf{S} = \text{diag}(0, \ldots, 0, \ldots, 0, \ldots, 0, \ldots, 0, \ldots), \]

\[ s_i = \frac{1}{c_i}, \quad s_0 = s_x = s_y = s_z = \frac{8}{8 - s_c}. \]

where \( c \) is the relaxation time:

\[ c = \frac{1}{2} + \frac{\mu}{\rho \Delta x \sqrt{R_e T}}. \]  

where \( \Delta x \) is the lattice length. Note that for high-Kn flow simulation the dynamic viscosity needs to be replaced by \( \mu_e \) using Eq. (32). The \( f_{\text{eq}} \) in Eq. (26) is the equilibrium distribution:

\[ f_{\text{eq}}^{(i)} = \omega_i \rho [1 + \frac{c_i u}{R_e T} + \frac{(c_i u)^2}{2(R_e T)^2} + \frac{u u}{2R_e T}], \]

where \( \omega_i \) is weight factor; \( \rho \) and \( u \) are local density and velocity respectively:

\[ \rho = \sum_i f_i, \quad u = \frac{1}{\rho} \sum_i f_i c_i. \]

To capture the gas slippage, we followed the scheme of Li et al., adopting the diffusive wall boundary and the effective viscosity correction:

\[ \mu_e = \frac{\mu}{1 + 2.2 K_n}. \]  

As for the inlet and outlet, the pressure boundary proposed by Guo et al. is adopted.

The multiscale method is validated by the simulation of non-slip ideal gas flow in a rigid slit. The downstream pressure \( p_2 \) is 0.1 MPa and the upstream pressure \( p_1 \) is 2, 4, 6 MPa, respectively. The other physical parameters and the LBM parameters are listed in Table 1. As shown in Fig. 4, the pressure profiles predicted by simulation agree well with the theoretical solutions:

\[ p^2 = -(p_1^2 - p_2^2) x/L + p_1^2. \]  

3.2. Validation of the characteristic pressure model

Three types of real gases and four stress sensitivities were chosen for simulation to validate the characteristic pressure model. The simulated mass flux is used in Eq. (15) and Eq. (17) to calculate the apparent permeability. The upstream pressure ranges from 0.5 to 6 MPa and other parameters are listed in Table 2. The mass flux results obtained from the simulation are given in the Appendix A.

First, the apparent permeability results calculated by Eq. (15) and Eq. (17) are compared, to examine the validity of the approximation Eq. (16). Fig. 5(a) presents the apparent permeability obtained from CO2 flow in slits with different stress sensitivities. Generally, the results of Eq. (15) and Eq. (17) are very close when \( \alpha \sigma_{\text{bar}} < 0.2 \). In a case with \( \alpha = 0.1 \) MPa\(^{-1}\) and \( \rho_{\text{bar}} = 2.3 \) MPa, the intrinsic permeability at upstream is 2.74 times as large as the value at downstream and \( \sigma/\gamma_l \) (MPa).  

Table 1

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slit height, L (nm)</td>
<td>100</td>
</tr>
<tr>
<td>Slit length, L (cm)</td>
<td>5</td>
</tr>
<tr>
<td>Temperature, T (°C)</td>
<td>45</td>
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</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>Gas type</td>
<td>CH4, CO2, C2H6</td>
</tr>
<tr>
<td>Stress sensitivity, ( \alpha ) (MPa(^{-1}))</td>
<td>0, 0.025, 0.05, 0.1</td>
</tr>
<tr>
<td>Reference slit height, ( l_0 ) (nm)</td>
<td>100</td>
</tr>
<tr>
<td>Slit length, L (cm)</td>
<td>5</td>
</tr>
<tr>
<td>Downstream pressure, ( p_2 ) (MPa)</td>
<td>0.1</td>
</tr>
<tr>
<td>Temperature, T (°C)</td>
<td>45</td>
</tr>
</tbody>
</table>
ranges from 1 to 1.96, but the error is only 1.6%, indicating the approximation Eq. (16) is acceptable. Fig. 5(a) also shows that a minimum permeability exists in Klinkenberg plot when the slit is stress-sensitive. The reason is that the poroelastic deformation enhances the flow at a high pore pressure while the gas slippage effect enhances the flow at a low pore pressure.

To further prove the accuracy of the characteristic pressure model, the deformation-corrected permeability \( K/\gamma K \) is plotted against \( Kn \) in Fig. 5(b). According to Eq. (4) and Eq. (8), \( K/\gamma K \) should be linear with \( Kn \):

\[
K(p)/\gamma_K(p) = K_{\infty,0}(1 + bKn) \quad (34)
\]

In Fig. 5(b), the predictions of the characteristic pressure model show a very good linear relationship. In addition when \( Kn \) approaches to 0, \( K/\gamma K \) recovers the theoretical value \( K_{\infty,0} = l_0/(12H) \). Thus, the characteristic pressure model is capable to accurately calculate the gas permeability of the real gas flow in deformable structures.

Next, the characteristic pressure model is tested for different gas types. The gas permeability is calculated by Eq. (15) and Eq. (17) with the simulated mass flow rates as inputs. The Klinkenberg plots for \( \text{CH}_4, \text{CO}_2 \) and \( \text{C}_2\text{H}_6 \) are presented in Fig. 6(a) and the \( K/\gamma K \) versus \( Kn \) is presented in Fig. 6(b). The results prove that the approximation Eq. (16) is acceptable and the characteristic pressure model is accurate for different gases. Comparing Fig. 5(a) with Fig. 6(a), we also find that the structural stress sensitivity mainly affects the high pressure region in the Klinkenberg plot, while the gas properties mainly affect the low pressure region. In the high pressure region, the poroelastic deformation dominates over the gas slippage effect, thus gas properties have a smaller impact on permeability (Fig. 6a). While in the low pressure region, the gas slippage effect dominates over the deformation, thus stress sensitivity becomes less important.

<table>
<thead>
<tr>
<th>Model</th>
<th>Representative work</th>
<th>Expression</th>
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<tr>
<td>Ideal gas model</td>
<td>Klinkenberg, 1941</td>
<td>( K(\sigma) = \frac{2p m L Q_{m}}{P_1 - P_2} \sigma R T ) (35)</td>
</tr>
<tr>
<td>Constant ( \mu ) at ( \sigma ) model</td>
<td>Gensterblum, 2014</td>
<td>( K(\sigma) = \frac{2p m L Q_{m}}{P_1 - P_2} \sigma R T ) (36)</td>
</tr>
<tr>
<td>Constant ( \mu ) and ( Z ) at ( \sigma ) model</td>
<td>Rushing, 2004</td>
<td>( K(\sigma) = \frac{2p m L Q_{m}}{P_1 - P_2} Z(\sigma, T) R T ) (37)</td>
</tr>
<tr>
<td>Real gas model</td>
<td>Present</td>
<td>( K(\rho_{\text{char}}) = \frac{l m_{\text{char}}}{P_1 - P_2} \int_{1/\rho_{\text{char}}}^{1/\rho_{\text{char}}} \frac{1}{Z(\sigma, T)} \sigma R T dp )</td>
</tr>
<tr>
<td>Characteristic pressure model</td>
<td>Present</td>
<td>( K(\rho_{\text{char}}) = \frac{l m_{\text{char}}}{P_1 - P_2} \int_{1/\rho_{\text{char}}}^{1/\rho_{\text{char}}} Z(\sigma, T) \sigma R T dp )</td>
</tr>
</tbody>
</table>
3.3. Model comparisons

The characteristic pressure model compared with the previous popular models for gas permeability calculations in Table 3. To distinguish the poroelastic effect and the real gas effect, we add the real gas model, which considers the real gas effect but neglects the deformation by keeping $\gamma_K = 1$ in Eq. (16).

Permeability is calculated by different models using the simulation results of CO$_2$ flow with the structural stress sensitivity $\alpha = 0.05$ MPa$^{-1}$. The Klinkenberg plot in Fig. 7(a) shows that the previous models tend to overestimate the apparent permeability due to non-consideration of the non-uniform deformation and the real gas effect. Fig. 7(a) also shows that the constant $\mu$ and $Z$ at $p$ model agrees well with the real gas model, which means that it can capture the real gas effect somehow. To further demonstrate the accuracy of the characteristic pressure model, the deformation-corrected permeability $K/\gamma_K$ versus $Kn$ is plotted in Fig. 7(b). Only the characteristic pressure model predicts the linear trend and recovers the theoretical value as $Kn$ approaches zero. Other models have large deviations when $Kn$ is small and predict a minimum of $K/\gamma_K$, which does not exist physically. The results prove that the characteristic pressure model is more reliable than other models when the poroelastic deformation and the real gas effect are strong.

4. Permeability predictions for experimental data

After validations, the characteristic pressure model was employed in laboratory experiments to calculate gas permeability using measured data. In the experiments, a shale sample and a coal sample from Poland were used. The steady-state experimental set-up and procedure are described in our previous works.$^{10,11}$ The conditions of the experiment are presented in Table 4, with varied upstream pressure from 0.5 MPa to 4 MPa. For each upstream pressure, the sample is measured three times and the fluctuation of mass flux is less than one percent (Appendix B). To better understand the pore system, we also determined the porosity and the pore size distribution of the shale sample. The porosity measured by He-pycnometry on the unstrressed sample was 9.8% and most pores had diameters less than 50 nm according to mercury intrusion porosimetry.

With the mass flux data, gas permeability coefficients were calculated by the four models (Table 3). To describe the poroelastic effect, we adopt the exponential empirical expression for intrinsic permeability$^{50}$:

$$K_m = K_{\infty,0} e^{-\beta(\mu, Z_p)}. \tag{38}$$

That yields:

$$\gamma_K = e^{-\beta(\mu, Z_p)}. \tag{39}$$

where $\beta$ represents the stress sensitivity of the porous medium. Since $\beta$ is determined using permeability data while the calculation of permeability also needs $\beta$, we first calculate permeability without considering the non-uniform deformation of the rock (i.e. assuming $\gamma_K$ is constant along the rock core). The measurements were performed at different confining pressures to obtain intrinsic permeability coefficients at different effective stress levels. Then $\beta$ values obtained by fitting these data with Eq. (38) are 0.165 MPa$^{-1}$ for the coal sample and 0.058 MPa$^{-1}$ for the shale sample. These values are applied in the characteristic pressure model to calculate the permeability. For simplicity, the $\beta$ is not calculated again using the new permeability data, thus its value is only an estimation.

The Klinkenberg plots predicted by different models are presented in Fig. 8. Fig. 8(a) shows the apparent permeability results obtained from methane flow in coal. Since coal is very stress-sensitive, the poroelastic deformation plays an important role and the permeability will be overestimated if the non-uniform deformation is neglected. Similar to the Fig. 7(a), Fig. 8(a) shows good agreements between the constant $\mu$ and $Z$ model and the real gas model, and the deviation between the characteristic pressure model and the previous models is mainly caused by the poroelastic deformation. Fig. 8(b) shows the apparent permeability results of CO$_2$ flow in shale. In this case, the real gas effect becomes more important than the deformation. Compared with the characteristic pressure model, we find the previous models using constant $\mu$ and $Z$ at $p$ can give acceptable predictions but the constant $\mu$ at $p$ model fails again.

The characteristic pressure model considers the poroelastic deformation and the real gas effects in the permeability calculation, which improves the accuracy of gas permeability obtained from laboratory measurements of tight rocks under large pressure gradient. This model provides theoretical support for the use of very high pressure gradients to produce measurable flow rates in laboratory measurements on ultra-tight rocks.

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Table 4

Parameters for the laboratory measurement of gas permeability.

<table>
<thead>
<tr>
<th>Core sample</th>
<th>Gas type</th>
<th>Coal</th>
<th>Shale</th>
</tr>
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<tbody>
<tr>
<td>Gas type</td>
<td>CH$_4$</td>
<td>CO$_2$</td>
<td></td>
</tr>
<tr>
<td>Sample length (m)</td>
<td>$3.33 \times 10^{-2}$</td>
<td>$1.30 \times 10^{-2}$</td>
<td></td>
</tr>
<tr>
<td>Sample cross section area (m$^2$)</td>
<td>$1.15 \times 10^{-3}$</td>
<td>$1.15 \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Confining pressure, $p_c$ (MPa)</td>
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<td>40</td>
<td></td>
</tr>
<tr>
<td>Downstream pressure, $p_d$ (MPa)</td>
<td>0.15</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Temperature, $T$ (°C)</td>
<td>23</td>
<td>45</td>
<td></td>
</tr>
</tbody>
</table>

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Fig. 7. Permeability coefficients calculated from CO$_2$ flow in a slit by different models: (a) Apparent permeability at reciprocal characteristic pressure (characteristic pressure model and real gas model) or reciprocal mean pressure (other models); (b) Apparent permeability $K$ over $\gamma_K$ versus $Kn$. The dotted line is the linear fitting for the characteristic pressure model (slope: $8.50 \times 10^{-15}$ m$^2$; intercept: $8.40 \times 10^{-17}$ m$^2$) and the red point is the theoretical value of $K_{\infty,0}$. 

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(a)  

(b)
5. Conclusions

We have developed an improved permeability model to determine the gas permeability of deep-seated tight rocks from laboratory steady-state measurements. Because a large pressure gradient is often required to drive the gas flow through tight rock samples, the non-uniform deformation and the real gas effects need to be carefully considered besides the slippage effect. A characteristic pressure model that considers all the three effects is developed, which predicts the apparent permeability at the characteristic pressure. The model is validated by independent numerical simulations using a multiscale hybrid scheme. In the simulations, we find that the poroelastic deformation dominates the high pressure region while the gas properties mainly influence the low pressure region. After validation, the characteristic pressure model has been applied to laboratory permeability measurements on coal and shale samples. Comparison with previous models suggests that the permeability may be overestimated if the deformation or the real gas effect is neglected. This model bridges laboratory measurements with reservoir engineering applications by (i) calculating more accurate permeability coefficients from experiments on tight rock cores and (ii) providing a theoretical support to use a high pressure gradient in laboratory experiments to make measurements more convenient and efficient.

Acknowledgements

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Appendix A

The simulation results are presented in this appendix. Table A1 gives the mass flux of gas flow in a slit between two elastic plates. Three types of gas and four different stress sensitivities are considered. The downstream pressure is kept at 0.1 MPa while the upstream pressure varies from 0.5 MPa to 10 MPa. The gas properties are shown in Fig. A1 and other physical conditions are listed in Table 2. The simulations are numerical experiments, which provide relationships between mass fluxes and pressure boundary conditions. The simulation results are used to evaluate the gas permeability models.

Table A1
Simulated mass flux $Q_m$ (kg m$^{-2}$ s$^{-1}$) of the gas flow between two elastic plates.

<table>
<thead>
<tr>
<th>$p_i$ (MPa)</th>
<th>CO$_2$</th>
<th>CH$_4$</th>
<th>C$_2$H$_6$</th>
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<tr>
<td>0</td>
<td>0.025</td>
<td>0.05</td>
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<table>
<thead>
<tr>
<th>$p_i$ (MPa)</th>
<th>CO$_2$</th>
<th>CH$_4$</th>
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Appendix B

This appendix presents mass flux data measured in our laboratory experiments (Table B1). Based on the data, apparent permeability coefficients are calculated using the gas permeability models (Fig. 8).

Table B1

<table>
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<tr>
<th>p_1 (MPa)</th>
<th>Q_m (kg m^{-2} s^{-1})</th>
<th>p_1 (MPa)</th>
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References


